A SPECIAL REPORT FROM

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Position Sizing Methods and their Role in Systematic Trading

Position Sizing and Systematic Trading

An integral step in developing a trading system is evaluating its performance. When we test a system on historical data, we are, in effect, simulating how that system will perform when we trade it. It only makes sense, then, that the more accurate the simulation, the better we can evaluate the system. If we could accurately account for all aspects of trading -- slippage, commissions, account size, number of contracts, market behavior, etc. -- then we would know just what to expect if we were to trade the system, and our decision about whether or not to trade the system would be better as a result.

Of course there's no 100% accurate way to simulate trading a system, but there are ways to increase the accuracy of the simulation. The most important way is through position sizing. Those of you, like me, who spend a lot of time working in TradeStation, are probably used to seeing the results of system testing expressed in terms of one-contract profits and losses. The TradeStation performance report is based on the assumption that all trades are the same size. For futures, this means each trade is typically one contract. For stocks, you would typically take 100 shares or some multiple of that for each trade.

TradeStation does allow for a variable number of shares or contracts for each trade. EasyLanguage makes it relatively simple to program just about any method you can think of to vary the number of contracts or shares for each trade. The problem is that most of the metrics in the TradeStation performance report lose their meaning if the number of contracts or shares is not fixed. Metrics such as the average trade, standard deviation of the average trade, the average win, average loss, maximum dollar drawdown, and net dollar profit become distorted and difficult to interpret if the number of contracts or shares varies from trade to trade. This is particularly true if you program rules to increase the position size as profits accumulate. In that case, many of the performance metrics, such as average trade and dollar drawdown, tend to increase with the number of trades.

Because TradeStation isn't designed to report performance results when using a variable number of contracts or shares, most traders probably stick to a uniform trade size when evaluating systems in TradeStation. What's wrong with that? In a nutshell, it makes it difficult to see the connection between risk and reward. We're all familiar with the concept that the greater the risk, the greater the reward. Higher profits are the compensation for taking on a greater risk. No one wants to assume more risk without being compensated for it. However, if you don't understand the true risk inherent in a system, then how do you know if you're being fairly compensated for it? More to the point, when comparing trading systems or when comparing parameter sets for a given system, we generally want to choose the one that produces the greatest reward for a given level of risk. In order to do this, we need to understand the risk-reward characteristics of our system.

As an example, consider the one-contract results from the following system:

Net profit: $5944
Number of trades: 487
Ave. trade: $12
Ave. winning trade: $28
Ave. losing trade: -$23
Percent profitable: 69.6%
Profit factor: 2.73
Largest loss: -$212
Max drawdown: -$1,573
Max consecutive losers: 4

These results are from a day trading system for the E-mini S&P 500 futures. Most people probably wouldn't trade this system because of its small average trade size. Also, notice that the largest loss is -$212. This comes from the fixed size stop used for each trade. This means that you're risking $212 on each trade to make $28. Not exactly a favorable risk-reward ratio. Put another way, with an average trade of $12, every time a trade is stopped out, the results of the past 17 trades are erased. On the other hand, the system has a high profit factor, and the drawdown is pretty reasonable for one contract. Assuming the average trade is achievable, would it make sense to try to scale this system up by trading more contracts or is the risk-reward ratio an insurmountable obstacle?

Position Sizing Can Help

An accurate simulation of this system taking into account position sizing, account equity, and margin requirements would answer this question. The type of position sizing I have in mind is based on risking a percentage of the trading account on each trade. For example, we might risk 3% of account equity on each trade. For the system above, if we had a $15,000 account, and the risk per contract is $212, risking 3% of the account would give us 0.03*15000/212 = 2.12 or 2 contracts. As the account equity grew, we'd be risking 3% of a larger number, which would give us more contracts. Added to a trading system simulation, this type of **position sizing allows us to relate risk to reward**.

If we ran the trade simulation for the system above assuming 3% of equity was risked on each trade, we could see what kind of drawdown we might expect, what kind of equity curve we might get, and what kind of returns to expect. We could try other risk percentages, too. If we did, we'd see that higher risk percentages give higher rates of return but higher drawdowns as well. By testing a number of different risk percentages, we could get a pretty good sense of the relationship between risk and return for this system. This is what I meant when I said that position sizing is a way to relate risk to reward.

You might have noticed that I'm using the word *risk* in two different ways. On the one hand, we use risk to refer to the amount of money or percentage of the trading account at risk on a particular trade. If the trade is a loss, we could lose $212, for example, or perhaps 3% of the trading account. This is the trade risk. On the other hand, the worst-case peak-to-valley drawdown of a trading system is a common and practical measure of the overall risk of a trading system. By risking a percentage of the account on each trade, the simulation can relate the trade risk to the drawdown risk as well as the rate of return to the drawdown risk.

Combining Position Sizing with Monte Carlo Analysis

A related method to improve the accuracy of trading system simulations is with Monte Carlo analysis. Inasmuch as maximum peak-to-valley drawdown is a useful measure of system risk, improving the calculation of the drawdown will improve our simulation results and thereby provide us with a better evaluation of the system. Although we can't predict how the market will differ tomorrow from what we've seen in the past, we do know it will be different. If we calculate the maximum drawdown based on the historical sequence of trades, we're basing our calculations on a sequence of trades we know won't be repeated exactly. Even if the distribution of trades (in the statistical sense) is the same in the future, the sequence of those trades is largely a matter of chance. Calculating the drawdown based on one particular sequence is somewhat arbitrary. Moreover, the sequence of trades has a very large effect on the calculated drawdown. If you choose a sequence of trades where five losses occur in a row, you could get a very large drawdown. The same trades arranged in a different order, such that the losses are evenly dispersed, might have a negligible drawdown.

As a way to address this problem, we can apply a Monte Carlo approach. The idea is to randomize the sequence of historical trades and calculate the rate of return and drawdown for the randomized sequence. We then repeat the process several hundred or thousand times. Looking at the results in aggregate, we might find, for example, that in 95% of the sequences, the drawdown was less than 30% when 4% of the equity was risked on each trade. We would interpret this to mean that there's a 95% chance that the drawdown will be less than 30% when 4% is risked on each trade.

Combining the Monte Carlo approach with risk-based position sizing improves our system trading simulations considerably. As an example, let's go back to the system results presented above. I took 200 consecutive trades from the system, spanning about 10 months. The risk for each trade was the same: $212. I started with an account size of $20,000. Running the trades through a Monte Carlo simulator produced the following table of results:

RESULTS AT 95% PROBABILITY

f value	Return(%)	Drawdown(%)
0.01	0	0
0.02	20.575	4.48405
0.03	39.475	6.80517
0.04	60.63	9.81719
0.05	84.6125	12.359
0.06	111.568	14.7371
0.07	141.488	17.3188
0.08	174.873	20.2822
0.09	211.705	22.2958
0.1	252.18	25.5526
0.11	295.582	26.8196
0.12	344.142	30.3462
0.13	397.335	31.8658
0.14	456.025	34.3499
0.15	518.737	37.4941
0.16	586.433	38.1433
0.17	661.447	42.035
0.18	740.9	44.3473
0.19	827.923	45.8333
0.2	921.35	46.3659
0.21	1021.49	47.9949
0.22	1128.46	51.2275

The first column, "f value" is the fraction of the account risked on each trade, also known as the **fixed fraction**. For example, 0.03 means that 3% is risked on each trade. The second column, "Return(%)", is the net rate of return on the starting equity over the period, and "Drawdown(%)" is the maximum (i.e., worst-case) peak-to-valley drawdown expressed as a percentage of the equity existing prior to the start of the drawdown. A drawdown of 20%, for example, means the account equity fell 20% from the highest equity peak preceding the drawdown. All calculations are on a closed trade basis. The results are tabulated at a confidence level of 95%.

This table provides an answer to the question posed earlier: would it make sense to try to scale this system up by trading more contracts or is the risk-reward ratio an insurmountable obstacle? Again, assuming the average one-contract trade size of $12 is achievable in practice, the Monte Carlo simulation suggests that, yes, this system is viable. Consider, for example, a fixed fraction of 0.06. By risking 6% of account equity on each trade, the Monte Carlo simulation estimates that the rate of return would be 111% and the worst-case drawdown would be about 15% with 95% confidence. As expected, we'd need to trade a fairly large number of contracts. A risk percentage of 6% implies that we'd be starting with 0.06 * 20000/212 = 5 contracts (rounded down). With an initial margin requirement of about $3500, this would require about $17,500 in initial margin. This would leave enough available equity to cover the position if it was stopped out at the maximum loss of $212. Even though the drawdown at the slightly higher risk percentage of 7% is only 17%, we would not be able to afford the number of contracts that this would require. The largest risk percentage that would work for this system is about 6%.

The Monte Carlo simulation is particularly useful for this system because it properly accounts for the unusual distribution of wins and losses in this system. While most of the trades in the system are small wins or small losses, there are periodic large wins and large losses. On a one-contract basis, the system produces very little profit. However, it's difficult to determine from the one-contract results if the relatively large risk represented by those periodic large losses would allow it to be scaled up to a large enough number of contracts to be viable. The Monte Carlo simulation takes all these factors into account to tell us that it should work.

While most systems are probably more straight-forward than the example presented here, the rationale for combining risk-based position sizing and Monte Carlo simulation is simple: the more accurately we can simulate the performance of a trading system, the better we can evaluate the system. And the better we can evaluate a system or a set of parameter values for a system, the better our trading is likely to be.

Fixed Ratio Position Sizing

In the preceding discussion of position sizing, I assumed that the number of contracts was determined based on the risk of the trade. For example, you might decide to risk 3% of your trading equity on the next trade. If the trade has a potential loss of $500 per contract, and your account equity is $50,000, you would take 3% of your equity (0.03 * $50,000 = $1,500) and divide the result by the trade risk of $500. The result is $1,500/$500 or 3 contracts. This is known as **fixed fractional position sizing** and is widely used in futures trading.

Basing position sizing on risk makes intuitive sense in that we know the greater the risk, the greater the reward. We expect that if we risk more of our equity on each trade, we will earn a higher return. This is the case with fixed fractional trading, provided we don't risk more than the so-called "optimal f" (see reference 1) and presuming our trading method is inherently profitable. Fixed fractional trading also helps us relate the risk of individual trades to the drawdown risk. By drawdown risk, I mean the largest percentage decline in equity from the most recent equity peak. Most traders have a limit to how much drawdown they can tolerate; e.g., 30%. By using the Monte Carlo simulation method discussed above, it's possible to relate the trade risk, as represented by the fixed fraction, to the drawdown risk.

However, fixed fractional position sizing is not the only method of position sizing available. I often get questions about fixed ratio position sizing, so in this section I'll discuss the concept of fixed ratio position sizing and compare it to the fixed fractional method. In his book "The Trading Game," (see reference 2) Ryan Jones introduced the fixed ratio method, which he developed to address some of the limitations he felt existed in fixed fractional position sizing.

The key concept of the fixed ratio method is the **delta**. The delta is the profit per contract needed to increase the number of contracts by one. For example, starting with one contract and with a delta of $5,000, you need a profit of $5,000 to increase the number of contracts to two. With two contracts, you need a profit per contract of $5,000 or $10,000 total from the two contracts to increase the number of contracts to three. With three contracts, you need a profit of $15,000 to increase the number of contracts to four. With four contracts, you need to profit of $20,000 to increase the number of contracts to five, and so on.

Based on the relationships presented by Jones, it's possible to derive the following equation for the number of contracts in fixed ratio position sizing:

N = 0.5 * [1 + (1 + 8 * Profit/delta)^0.5]

where Profit = total closed trade profit in dollars, delta = profit/contract to increase by one contract, and "^0.5" means that the expression in parentheses is raised to the power of 0.5.

It's interesting to compare this equation to the corresponding equation for fixed fractional trading:

$$N = ff * Equity/| \text{trade risk} |$$

where "trade risk" is the possible loss in dollars for the trade, and the vertical bars (|) represent absolute value. Notice that the relationship between the number of contracts and the profit is linear with fixed fractional trading. As the profits accrue, the number of contracts increases linearly. The rate of change of N with respect to account equity is constant with the fixed fractional method; e.g., a $10,000 increase in profits results in the same increase in the number of contracts regardless of whether that profit occurred with a $15,000 account or a $150,000 account.

With fixed ratio trading, on the other hand, as you accrue more profits, the number of contracts increases more slowly. A $10,000 profit with a $20,000 account will increase the number of contracts more than if a $10,000 profit is made on a $200,000 account. For small account sizes, you'll increase the number of contracts more quickly with fixed ratio position sizing. However, when the account equity becomes larger, the number of contracts will increase more slowly than with fixed fractional position sizing. This is why fixed ratio position sizing is sometimes preferred for small accounts.

Because the fixed ratio method depends on account size, how it performs compared to the fixed fractional method over a series of trades depends on where the drawdowns occur. If the biggest drawdown occurs late in the sequence of trades, the fixed ratio method will do well because the fastest increase in the number of contracts will have occurred during the most profitable period. On the other hand, if the biggest run-up in equity occurs late in the sequence of trades, the fixed fractional method will do better because it will increase the number of contracts more quickly at that point, whereas the rate of increase in the number of contracts with the fixed ratio method will have already slowed.

As an example, consider the following equity curve from a real sequence of trades. I adjusted the delta for the fixed ratio method and the fixed fraction for the fixed fractional method so that the worst-case percentage drawdown was the same in each case. Fig. 1 shows the equity curves for both methods when the trades occur in their historical sequence. The fixed ratio method clearly delivers superior performance. The net profit is much higher for the same maximum drawdown. Note that the primary run up in equity occurred early in the sequence of trades. The fixed ratio method was more aggressive early on when it mattered the most.

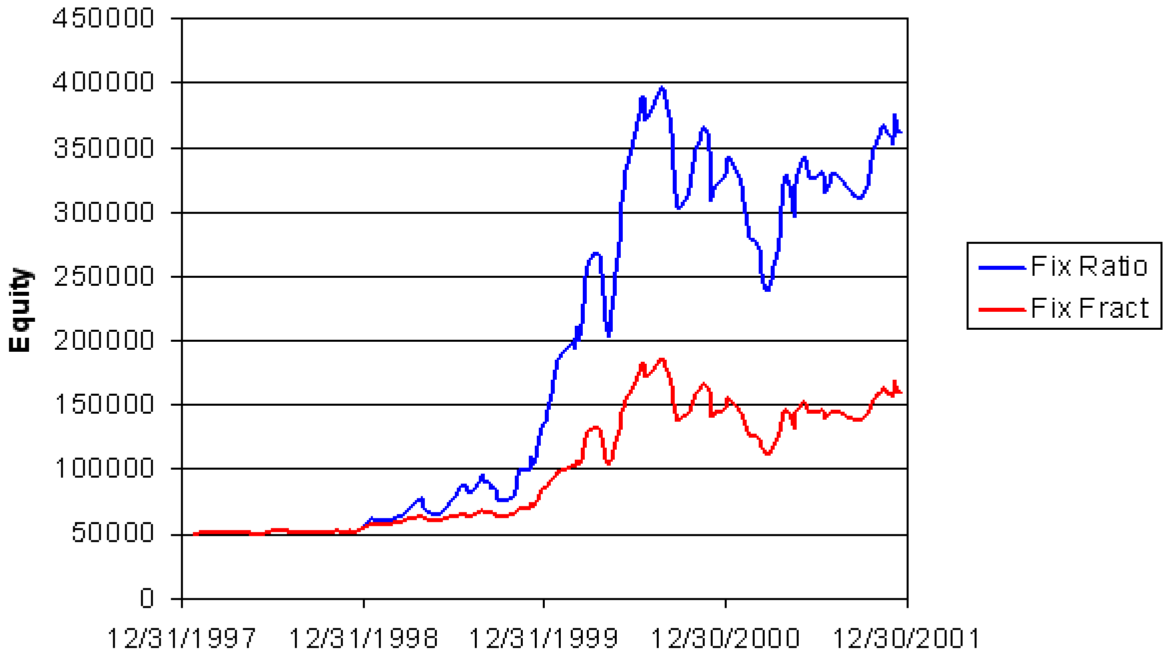

Fig. 1. Equity curve for historical sequence of trades using fixed fractional ("Fix Fract") and fixed ratio ("Fix Ratio") position sizing. The maximum peak-to-valley drawdown is the same (in percentage terms) in each case.

However, if we randomize the trade sequence, as in Fig. 2, the opposite result is possible. These are the same trades as in Fig. 1, just in a different order. Again, the parameters for the two methods have been adjusted to produce the same maximum peak-to-valley percentage drawdown. In this case, the fixed fractional method generates a much higher return for the same drawdown. In this sequence of trades, the run-up in equity occurred late in the sequence. As a result, the fixed fractional method was more aggressive than the fixed ratio method in increasing the number of contracts late in the sequence when it counted most.

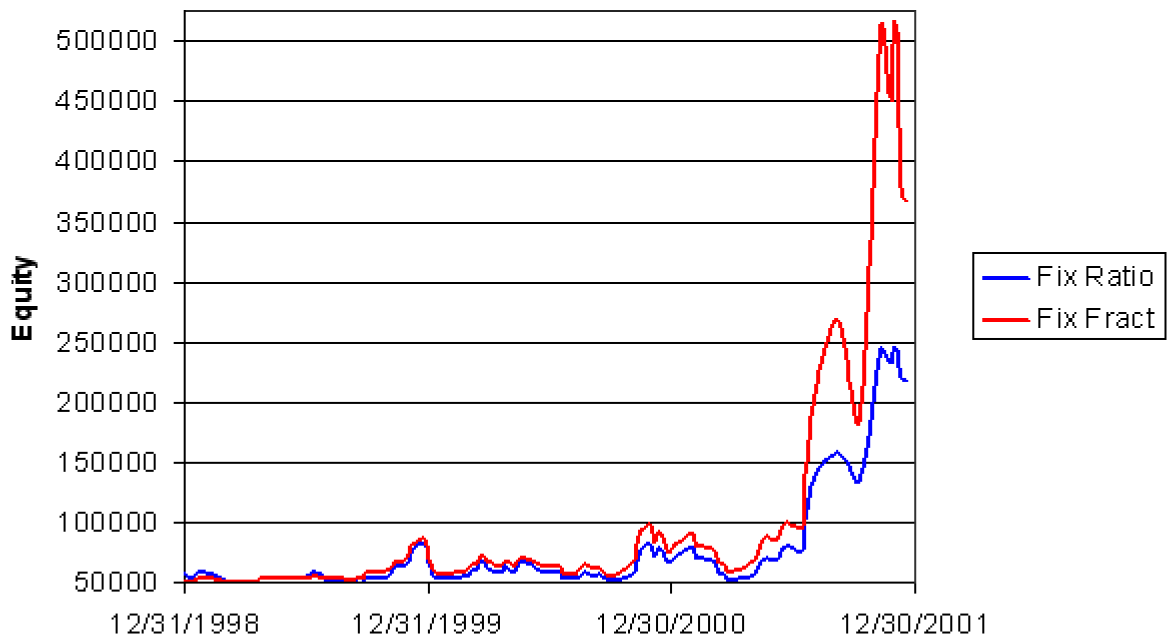

Fig. 2. Equity curve for randomized sequence of the same trades as in Fig. 1 using fixed fractional ("Fix Fract") and fixed ratio ("Fix Ratio") position sizing. The maximum peak-to-valley drawdown is the same (in percentage terms) in each case.

For any sequence of trades, one method will be better than the other. However, even if we have a good idea of the *distribution* of our trades, the sequence is always unknown. As I discussed in the previous section, one way to deal with the consequences of not knowing the sequence of a series of trades is to use the Monte Carlo method. With the Monte Carlo method, we can perform an analysis over many different, randomly chosen trade sequences and evaluate the results in terms of statistics. In effect, this is a way to convert the uncertainty of the trade sequence into a quantified (if probabilistic) result. This method might be able to tell us whether the fixed fractional or fixed ratio method is better for a given series of trades.

We can take this analysis one step further by reconsidering the equation presented above for the fixed ratio method. Notice the 0.5 exponent in the equation for the number of contracts, N, in the fixed ratio method. Consider what we would get if the 0.5 was replaced with 0. In that case, we get N = 1. In other words, an exponent of zero represents fixed contract trading with one contract per trade. What if the exponent has the value 1? In this case, we find that the number of contracts, N, is proportional to the profit. This is the basis of fixed fractional trading. In other words, an exponent of 1 represents fixed fractional position sizing.

There's nothing preventing us from choosing other exponent values as well. With this in mind, we can write a more generalized form of the position sizing equation as:

N =   0.5 * [1 + (1 + 8 * Profit/delta)^m]

where the exponent m can vary from 0 to any positive number we like. With m = 0, we get fixed contract trading. With m = 1, we have the equivalent of fixed fractional trading. m = 0.5 gives us fixed ratio trading.

Any value of m less than 1 (e.g., 0.5 or 0.10) will increase the number of contracts more slowly for larger account equity values. Values of m larger than 1 will increase the number of contracts more quickly as the account equity increases. At m = 1 (i.e., fixed fractional), the rate of change in the number of contracts is independent of account size.

We might expect that for any sequence of trades, there's an "optimal" value of m. By optimal, I mean there's one value of m that produces the greatest return for a given maximum drawdown. As noted above, since we don't know the sequence of trades to expect in the future, calculating this optimal m for a historical sequence of trades is probably a pointless exercise. However, it might be interesting to use the Monte Carlo method to see what this optimal m would be based on the statistical results of the Monte Carlo simulation.

References:

1. Ralph Vince, Portfolio Management Formulas, John Wiley & Sons, Inc., New York, 1990.
2. Ryan Jones, The Trading Game, John Wiley & Sons, Inc., New York, 1999.

About the Author:

Michael Bryant is the owner of Adaptrade Software and Breakout Futures. He holds degrees in engineering from the University of Connecticut (BS) and the University of Wisconsin-Madison (MS, PhD). He has been trading and studying the futures markets since 1994 and has developed Market System Analyzer (MSA), a software program for applying position sizing and money management analysis to trading strategies. Please visit **www.Adaptrade.com** to contact Dr. Bryant or to comment on this report.